Stability of Texture Zeros under Radiative Corrections in See-Saw Models

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Abstract

It has been shown that only certain neutrino mass matrices with texture zeros are compatible with existing data. We discuss the stability of phenomenological consequences of texture zeros under radiative corrections in the type-I see-saw scenario. We show that under certain conditions additional patterns are allowed due to these effects.

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1   Introduction

Remarkable progress has been made in recent years in the determination of neutrino masses and mixings, especially due to the results from oscillation experiments. The fact that two of the three mixings turned out to be large came as a surprise and shows that we are still far from a fundamental understanding of the flavour structure of the Standard Model (SM) and its extensions which include neutrino masses. This surprise should remind us that the absolute neutrino masses need not be hierarchical as naïvely expected from the analogy to the quark sector. This is important, since renormalization group (RG) effects can then have a considerable impact on the evolution of flavour structures towards the scale where they are explained.

Similar to models in the quark sector [1–9], one framework to understand the neutrino masses and mixings is to assume certain elements of the mass matrix to be zero [10–23]. Such texture zeros can be produced, for example, by flavour symmetries, see e.g. [24]. However, these symmetries are often broken at a high energy scale, so that RG running destroys the texture zeros. A classification of textures with two or more zeros and their compatibility with experimental data was given in refs. [10, 11]. We will study how RG corrections affect these results. In particular, we will show that some of the patterns which have been excluded become allowed, if the running leads to sufficiently large entries in the positions of the former zeros.

The running of neutrino parameters is described at low energies by the RG equations for the leading neutrino mass operator. In the SM and its minimal supersymmetric extension (MSSM) with massive neutrinos, this is a dimension 5 operator, whose energy dependence is determined by the RG equation [25–28]

$$16\pi^2 \frac{\mathrm{d}\kappa}{\mathrm{d}t} = C \, (Y_e^\dagger Y_e)^T \, \kappa + C \, \kappa \, (Y_e^\dagger Y_e) + \alpha \, \kappa \,. \tag{1}$$

The neutrino mass matrix is defined as $m_\nu = \frac{v^2}{4}\kappa$, where $v \approx 246$ GeV in the SM and $v \approx \sin\beta \cdot 246$ GeV in the MSSM. The renormalization scale μ enters as usual via $t = \ln(\mu/\mu_0)$, and $C = -\frac{3}{2}$ in the SM, $C = 1$ in the MSSM. Finally, α is a flavour scalar containing e.g. gauge couplings. Hence, the last term in the RG equation is flavour-blind and leads only to an overall rescaling which does not affect the mixing parameters.

In this paper, we work in the basis where the Yukawa matrix of the charged leptons Y_e is diagonal, i.e. we use the same classification of textures as in [10, 11]. From eq. (1) it follows then immediately that the radiative corrections to each element of the neutrino mass matrix (or, equivalently, κ) are proportional to this element itself, so that a zero entry remains zero. However, this changes in the type-I see-saw scenario [29–32] at energies larger than the masses of the heavy singlet neutrinos that are introduced in order to explain the smallness of the light neutrino masses. Above this threshold, the neutrino Yukawa couplings Y_ν contribute to the RG equations. In general, they are non-diagonal and hence cause a mixing of the mass matrix elements, so that the elements which are zero at the high-energy scale obtain a finite value at low energies and thus texture zeros are destroyed. We will determine under which conditions this modifies the compatibility of textures with experimental data. We will apply a bottom-up approach, i.e. we will study if the RG running from low to high energies can lead to a zero element. Therefore, we will refer to this possibility as radiative generation rather than destruction of texture zeros.

2 Conditions for the Stability of Textures

2.1 Running of the Effective Neutrino Mass Matrix

In the full theory with singlet neutrinos, i.e. above the highest see-saw scale, the relevant quantity for our discussion is the effective light neutrino mass matrix defined as

$$m_\nu = -\frac{v^2}{2} Y_\nu^T M^{-1} Y_\nu \; , \tag{2}$$

where M is the mass matrix of the heavy singlets and v is the relevant Higgs vacuum expectation value at low energy. From the RG equations for Y_ν and M [33–38] one obtains

$$16\pi^2 \frac{\mathrm{d}m_\nu}{\mathrm{d}t} = (C_e \, Y_e^\dagger Y_e + C_\nu \, Y_\nu^\dagger Y_\nu)^T \, m_\nu + m_\nu \, (C_e \, Y_e^\dagger Y_e + C_\nu \, Y_\nu^\dagger Y_\nu) + \alpha' \, m_\nu \tag{3}$$

with $C_e = -\frac{3}{2}$, $C_\nu = \frac{1}{2}$ in the SM, $C_e = C_\nu = 1$ in the MSSM, and

$$\alpha'_{\mathrm{SM}} = -\frac{9}{10} g_1^2 - \frac{9}{2} g_2^2 + 2 \operatorname{Tr}(Y_\nu^\dagger Y_\nu) + 2 \, (y_\tau^2 + y_\mu^2 + y_e^2) +$$
$$+ 6 \, (y_t^2 + y_b^2 + y_c^2 + y_s^2 + y_d^2 + y_u^2) \; , \tag{4a}$$

$$\alpha'_{\mathrm{MSSM}} = -\frac{6}{5} g_1^2 - 6 g_2^2 + 2 \operatorname{Tr}(Y_\nu^\dagger Y_\nu) + 6 \, (y_t^2 + y_c^2 + y_u^2) \; , \tag{4b}$$

where y_i are Yukawa couplings. We use GUT charge normalization for the U(1) gauge coupling. Between the mass thresholds, the running is modified because the singlets are successively integrated out [39]. Note that the RG equations for the effective mass operators cannot be obtained from eq. (3) by omitting suitable Yukawa couplings in general [40].

2.2 Mass Matrix Elements

In the bottom-up approach, one can understand under which circumstances the radiative generation of texture zeros is possible. If an element of the neutrino mass matrix, say $m_{\nu_{ij}}$, is to be zero at the GUT scale M_{GUT}, the sum of its value at the see-saw scale M_1 and its change due to the running between this scale and M_{GUT} has to be zero. In linear approximation, this means

$$m_{\nu_{ij}}\big|_{M_1} \approx -\dot{m}_{\nu_{ij}}\big|_{M_1} \cdot \ln \frac{M_{\mathrm{GUT}}}{M_1} \; . \tag{5}$$

If $m_{\nu_{ij}}$ is complex, this relation must be satisfied for both its real and its imaginary part. Hence, we can assume m_ν to be real in the following derivation if we keep in mind that each equation actually stands for two. We also assume that all singlet neutrinos are integrated out at the mass M_1 of the lightest singlet. To obtain a conservative bound, we choose this mass to be larger than about 10^9 GeV. Of course, smaller values are possible in principle, but then the neutrino Yukawa couplings have to be rather small as well, so that the RG evolution is actually suppressed. In the numerical examples presented later on, the see-saw scale lies comfortably above this lower bound. With $M_1 \gtrsim 10^9$ GeV and $M_{\mathrm{GUT}} \sim 10^{16}$ GeV, eq. (5) becomes

$$-\frac{16\pi^2 \, \dot{m}_{\nu_{ij}}}{m_{\nu_{ij}}} \gtrsim 10 \; . \tag{6}$$

Here and in the following, we implicitly assume the values of energy-dependent quantities like m_ν to be taken at M_1. Keeping only the top and tau Yukawa couplings and introducing the abbreviation $H := Y_\nu^\dagger Y_\nu$, the RG equation (3) can be rewritten for an individual matrix element,

$$16\pi^2 \dot{m}_{\nu_{ij}} = (m_\nu H)_{ij} + (m_\nu H)_{ji} + \sigma m_{\nu_{ij}} \tag{7}$$

with

$$\sigma := 2 \operatorname{Tr} H + (\delta_{i3} + \delta_{j3}) y_\tau^2 + 6 y_t^2 - \frac{6}{5} g_1^2 - 6 g_2^2 \stackrel{M_1 = 10^{13} \text{ GeV}}{\approx} 2 \operatorname{Tr} H + (\delta_{i3} + \delta_{j3}) y_\tau^2 + 1 \tag{8}$$

in the MSSM. Note that σ is always positive. Plugging (7) into ineq. (6) and collecting all terms independent of m_ν on the r.h.s. yields

$$-\frac{\sum_{k \neq j} m_{\nu_{ik}} H_{kj} + \sum_{k \neq i} m_{\nu_{jk}} H_{ki}}{m_{\nu_{ij}}} \gtrsim 10 + \sigma + H_{ii} + H_{jj} . \tag{9}$$

Careful inspection of this relation shows that radiative generation of texture zeros requires at least one element of m_ν to be roughly an order of magnitude larger than the value of the zero candidate at the see-saw scale. In the complex case, both the real and the imaginary parts of the matrix elements have to satisfy this requirement.

To see this, consider the easiest case first: let us assume that the l.h.s. of ineq. (9) is dominated by a single term, say $m_{\nu_{ik}} H_{kj}$ ($i \neq j$), and that $H_{kj} \sim 1$. Then $H_{jj} \sim 1$, H_{ii} is small, and $\operatorname{Tr} H \sim 2$. Hence, we obtain

$$-\frac{m_{\nu_{ik}}}{m_{\nu_{ij}}} \gtrsim 16 . \tag{10}$$

In principle, H_{kj} could be as large as 3 as long as all $|Y_{\nu_{ij}}| \lesssim 1$. This would reduce the required hierarchy somewhat, but the difference is not dramatic as the terms containing H on the r.h.s. become larger as well.

The smallest hierarchy is expected if all elements of H are large. For $H_{ij} \sim 1$, we find

$$-\frac{\sum_{k \neq j} m_{\nu_{ik}} + \sum_{k \neq i} m_{\nu_{jk}}}{m_{\nu_{ij}}} \gtrsim 19 , \tag{11}$$

so that

$$-\frac{m_{\nu_{ik}}}{m_{\nu_{ij}}} \gtrsim 5 \tag{12}$$

if all elements of the neutrino mass matrix except $m_{\nu_{ij}}$ are approximately equal. Thus, the required hierarchy can be a bit smaller than an order of magnitude. However, in this case it is difficult to find a Yukawa matrix which is perturbative and still reproduces the correct neutrino mass parameters via the see-saw formula. Hence, we conclude that the above statement about the required hierarchy in m_ν is reasonably conservative.

In the SM, the first two terms on the r.h.s. of eq. (7) are multiplied by an additional factor of $\frac{1}{2}$, and σ is replaced by

$$\sigma = 2 \operatorname{Tr} H + 6 y_t^2 - \frac{9}{10} g_1^2 - \frac{9}{2} g_2^2 \stackrel{M_1 = 10^{13} \text{ GeV}}{\approx} 2 \operatorname{Tr} H + 0.4 , \tag{13}$$

where we have neglected the tau Yukawa coupling, too, since it is always small in the SM. Because of the change in eq. (7), the term $10 + \sigma$ in ineq. (9) has to be multiplied by 2. Thus, a hierarchy is necessary in the SM as well, and it has to be even larger than in the MSSM, so that the generation of texture zeros becomes harder. On the other hand, the effects of the running between the mass scales of the heavy singlets can be especially significant in the SM [40] and might provide a loophole attenuating the restrictions found in this section.

2.3 Mass Eigenvalues

Let us now see what the hierarchy requirement for the elements of the neutrino mass matrix means for the mass eigenvalues. For this purpose, we express the matrix elements in terms of the eigenvalues m_1, m_2, m_3 and the mixing parameters. We use the approximations $\theta_{13} \approx 0$ and $\Delta m_\odot^2 \ll \Delta m_a^2$. For a normal mass hierarchy, we have $m_1 := m$, $m_2 = \sqrt{m^2 + \Delta m_\odot^2}$ and $m_3 \approx \sqrt{m^2 + \Delta m_a^2}$. Using

$$U^T m_\nu U = \text{diag}(m_1, m_2, m_3) \tag{14}$$

and the standard parameterization for the lepton mixing matrix U in the limit $\theta_{13} = 0$,

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -c_{23}s_{12} & c_{23}c_{12} & s_{23} \\ s_{23}s_{12} & -s_{23}c_{12} & c_{23} \end{pmatrix} \cdot \text{diag}(e^{-i\varphi_1/2}, e^{-i\varphi_2/2}, 1) \,, \tag{15}$$

we obtain (assuming without loss of generality that all unphysical phases in U are zero)

$$m_{\nu_{11}} \approx \sqrt{m^2 + \Delta m_\odot^2}\, s_{12}^2\, e^{i\varphi_2} + m\, c_{12}^2\, e^{i\varphi_1} \,, \tag{16a}$$

$$m_{\nu_{22}} \approx \sqrt{m^2 + \Delta m_a^2}\, s_{23}^2 + \sqrt{m^2 + \Delta m_\odot^2}\, c_{12}^2 c_{23}^2\, e^{i\varphi_2} + m\, s_{12}^2 c_{23}^2\, e^{i\varphi_1} \,, \tag{16b}$$

$$m_{\nu_{33}} \approx \sqrt{m^2 + \Delta m_a^2}\, c_{23}^2 + \sqrt{m^2 + \Delta m_\odot^2}\, c_{12}^2 s_{23}^2\, e^{i\varphi_2} + m\, s_{12}^2 s_{23}^2\, e^{i\varphi_1} \,, \tag{16c}$$

$$m_{\nu_{12}} \approx \left(\sqrt{m^2 + \Delta m_\odot^2}\, e^{i\varphi_2} - m\, e^{i\varphi_1} \right) c_{12}s_{12}c_{23} \,, \tag{16d}$$

$$m_{\nu_{13}} \approx -\left(\sqrt{m^2 + \Delta m_\odot^2}\, e^{i\varphi_2} - m\, e^{i\varphi_1} \right) c_{12}s_{12}s_{23} \,, \tag{16e}$$

$$m_{\nu_{23}} \approx \left(\sqrt{m^2 + \Delta m_a^2} - \sqrt{m^2 + \Delta m_\odot^2}\, c_{12}^2\, e^{i\varphi_2} - m\, s_{12}^2\, e^{i\varphi_1} \right) c_{23}s_{23} \,, \tag{16f}$$

where $s_{12} := \sin\theta_{12}$, $c_{12} := \cos\theta_{12}$ etc. If the mass spectrum is hierarchical, i.e. $m \approx 0$, the elements of the first row and column of m_ν are of the order of $\sqrt{\Delta m_\odot^2}$, while the others are of the order of $\sqrt{\Delta m_a^2}$. Hence, the maximal hierarchy between two elements of m_ν is roughly $\sqrt{\Delta m_\odot^2/\Delta m_a^2} \sim 1/6$, which is not enough for the generation of a texture zero. This statement holds independent of the Majorana phases φ_1 and φ_2, since each matrix element is dominated by just a single term. Note that the running of the mixing angles between the electroweak and the see-saw scale does not change this conclusion, since it is not significant for a strong normal hierarchy [41–43].

For quasi-degenerate neutrinos, i.e. $m^2 \gg \Delta m_{\mathrm{a}}^2$, the above formulae simplify to

$$m_{\nu_{11}} \approx m \left(s_{12}^2 \, e^{i\varphi_2} + c_{12}^2 \, e^{i\varphi_1} \right) , \tag{17a}$$

$$m_{\nu_{22}} \approx m \left(s_{23}^2 + c_{12}^2 c_{23}^2 \, e^{i\varphi_2} + s_{12}^2 c_{23}^2 \, e^{i\varphi_1} \right) , \tag{17b}$$

$$m_{\nu_{33}} \approx m \left(c_{23}^2 + c_{12}^2 s_{23}^2 \, e^{i\varphi_2} + s_{12}^2 s_{23}^2 \, e^{i\varphi_1} \right) , \tag{17c}$$

$$m_{\nu_{12}} \approx m \left(e^{i\varphi_2} - e^{i\varphi_1} \right) c_{12} s_{12} c_{23} , \tag{17d}$$

$$m_{\nu_{13}} \approx -m \left(e^{i\varphi_2} - e^{i\varphi_1} \right) c_{12} s_{12} s_{23} , \tag{17e}$$

$$m_{\nu_{23}} \approx m \left(1 - c_{12}^2 \, e^{i\varphi_2} - s_{12}^2 \, e^{i\varphi_1} \right) c_{23} s_{23} . \tag{17f}$$

As the situation now depends on the values of the Majorana phases, we consider the real cases first. For $\varphi_1 = \varphi_2 = 0$, the diagonal elements of the mass matrix approach m, while the off-diagonal entries go to zero. One can show that $m_{\nu_{ij}}/m_{\nu_{ii}}$ $(i \neq j)$ are monotonous functions of m. Hence, for a sufficiently large neutrino mass scale the required hierarchy in the matrix elements arises and the radiative generation of zeros in the off-diagonal elements becomes possible. In contrast, it is not possible to generate zeros in the diagonal entries.

If one Majorana phase is π and the other one zero, i.e. one of the mass eigenstates has a negative CP parity, all $|m_{\nu_{ij}}|$ become large for $m \to \infty$. The smallest entry is $|m_{\nu_{11}}| \approx m \cos 2\theta_{12}$, but it cannot be much smaller than the other matrix elements due to the experimental bound $\cos 2\theta_{12} > 0.22$ at the 3σ level [44]. Quantum corrections between the electroweak and the see-saw scale are not likely to change this situation, since they are not significant in the SM, and since they only cause a decrease of θ_{12} in the MSSM [45] unless θ_{13} is close to its experimental upper limit [43]. Consequently, the generation of a texture zero is unlikely.

The last real case, $\varphi_1 = \varphi_2 = \pi$, leads to the same conclusions for the first row and column as $\varphi_1 = \varphi_2 = 0$, because only the phase difference is relevant here. The situation changes for the diagonal elements $m_{\nu_{22}}$ and $m_{\nu_{33}}$, which go to $\pm m \cos 2\theta_{23}$ now, so that zeros can be generated in these elements if θ_{23} is close to $45°$. The remaining off-diagonal entry $m_{\nu_{23}}$ stays large for large atmospheric mixing and thus cannot be driven to zero by the RG in this case.

For a complex mass matrix, i.e. arbitrary phases, the hierarchy requirement must be satisfied by both the real and the imaginary parts of the respective matrix elements, as mentioned in the beginning of this section. In the 23-block of m_ν, this is not possible even if the Majorana phases are very small, because the hierarchy in the real cases is due to a cancellation between the three terms contributing to $m_{\nu_{23}}$, only two of which contribute to the imaginary part. Consequently, one expects at first sight that texture zeros can only be generated radiatively in the CP-conserving cases. However, this is too pessimistic, since adding a small imaginary part to the neutrino mass matrix does not change the mixing angles and mass squared differences significantly, except for special cases such as exactly degenerate mass eigenvalues. This allows us to make the imaginary part vanish by slightly adjusting the initial mass matrix, if the values of the Majorana phases are not too far away from 0 or π. For the elements of the first row and column, only the difference of the phases is relevant. Hence, the radiative generation of a texture zero in these positions is possible for arbitrary Majorana phases, provided that their difference is small.

If θ_{13} is non-zero, a finite Dirac phase δ could be an obstacle for the generation of texture zeros as well. However, as θ_{13} is experimentally restricted to be relatively small

and does not grow too much under the RG [43], the same argument holds as for small Majorana phases, so that the value of δ is not very important.

In the case of an inverted mass hierarchy, we use the convention $m_3 := m$, $m_2 = \sqrt{m^2 + |\Delta m_\text{a}^2|}$ and $m_1 = \sqrt{m^2 + |\Delta m_\text{a}^2| - \Delta m_\odot^2} \approx m_2$. Then the elements of the neutrino mass matrix can be expressed as

$$m_{\nu_{11}} \approx \sqrt{m^2 + |\Delta m_\text{a}^2|} \left(s_{12}^2 e^{i\varphi_2} + c_{12}^2 e^{i\varphi_1} \right), \tag{18a}$$

$$m_{\nu_{22}} \approx \sqrt{m^2 + |\Delta m_\text{a}^2|} \, c_{23}^2 \left(c_{12}^2 e^{i\varphi_2} + s_{12}^2 e^{i\varphi_1} \right) + m \, s_{23}^2, \tag{18b}$$

$$m_{\nu_{33}} \approx \sqrt{m^2 + |\Delta m_\text{a}^2|} \, s_{23}^2 \left(c_{12}^2 e^{i\varphi_2} + s_{12}^2 e^{i\varphi_1} \right) + m \, c_{23}^2, \tag{18c}$$

$$m_{\nu_{12}} \approx \left(\sqrt{m^2 + |\Delta m_\text{a}^2|} \, e^{i\varphi_2} - \sqrt{m^2 + |\Delta m_\text{a}^2| - \Delta m_\odot^2} \, e^{i\varphi_1} \right) c_{12} s_{12} c_{23}, \tag{18d}$$

$$m_{\nu_{13}} \approx -\left(\sqrt{m^2 + |\Delta m_\text{a}^2|} \, e^{i\varphi_2} - \sqrt{m^2 + |\Delta m_\text{a}^2| - \Delta m_\odot^2} \, e^{i\varphi_1} \right) c_{12} s_{12} s_{23}, \tag{18e}$$

$$m_{\nu_{23}} \approx \left(-\sqrt{m^2 + |\Delta m_\text{a}^2|} \left(c_{12}^2 e^{i\varphi_2} + s_{12}^2 e^{i\varphi_1} \right) + m \right) c_{23} s_{23} \tag{18f}$$

for $\theta_{13} \approx 0$. We have not used the approximation $m_1 \approx m_2$ for $m_{\nu_{12}}$ and $m_{\nu_{13}}$ to avoid underestimating the size of these entries for small m and equal Majorana phases. For the other matrix elements, this approximation is sufficiently accurate, since the mass eigenvalues are multiplied by s_{12}^2 and c_{12}^2, respectively. Experimentally, these numbers are known to be unequal (and this does not change during the running up to the see-saw scale, as mentioned above), so that there is no complete cancellation if $\Delta m_\odot^2$ is neglected.

A strong mass hierarchy causes most elements of m_ν to be of the order of $\sqrt{\Delta m_\text{a}^2}$, so that the radiative generation of texture zeros in these positions is not possible. The exceptions are $m_{\nu_{12}}$ and $m_{\nu_{13}}$, which are smaller than the other elements by a factor of about $\Delta m_\odot^2 / \Delta m_\text{a}^2$ for equal Majorana phases. Hence, these entries can become zero at the GUT scale even for relatively small values of m.

For quasi-degenerate masses, the conclusions are the same as for a normal mass ordering. This is not surprising as the only difference is the sign of the atmospheric mass squared difference, which is not important for $|\Delta m_\text{a}^2| \ll m^2$.

An overview of the results of this section is given in table 1. The positions in the neutrino mass matrix where a texture zero can be generated radiatively are marked by a "∘". Comparing these results with the classification of two-zero textures in the literature, we find that three of the six forbidden textures in [11] can be reconciled with data by the RG evolution. These are the patterns of class F,

$$\begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}, \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix},$$

where the crosses "×" stand for the non-zero entries. On the other hand, class E, i.e. the matrices of the form

$$\begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}, \begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}, \begin{pmatrix} 0 & \times & \times \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix},$$

remain forbidden. The reason for this is the zero in the 11-element, which cannot be generated radiatively due to the large deviation of the solar mixing angle from $\pi/4$ (cf.

Neutrino masses	Majorana phases	
Normal hierarchy, $m_1 \approx 0$	arbitrary	$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$
Inverted hierarchy, $m_3 \approx 0$	$\varphi_1 \approx \varphi_2$	$\begin{pmatrix} \cdot & \circ & \circ \\ \circ & \cdot & \cdot \\ \circ & \cdot & \cdot \end{pmatrix}$
	$\varphi_1 \not\approx \varphi_2$	$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$
Quasi-degenerate	$\varphi_1 \approx \varphi_2 \approx 0$	$\begin{pmatrix} \cdot & \circ & \circ \\ \circ & \cdot & \circ \\ \circ & \circ & \cdot \end{pmatrix}$
	$\varphi_{1,2} \approx 0,\ \varphi_{2,1} \approx \pi$	$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$
	$\varphi_1 \approx \varphi_2 \approx \pi$	$\begin{pmatrix} \cdot & \circ & \circ \\ \circ & \circ & \cdot \\ \circ & \cdot & \circ \end{pmatrix}$
	$\varphi_1 \approx \varphi_2 \not\approx 0, \pi$	$\begin{pmatrix} \cdot & \circ & \circ \\ \circ & \cdot & \cdot \\ \circ & \cdot & \cdot \end{pmatrix}$
	$\varphi_1 \not\approx \varphi_2$	$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

Table 1: Possible positions of radiatively generated texture zeros in the neutrino mass matrix, marked by a "∘". For $\varphi_1 \approx \varphi_2 \approx \pi$, at most 3 of the 4 zeros can be produced at the same time.

eqs. (17a), (18a)). One could hope to circumvent this problem by assuming that this texture zero exists already at low energies and that only the second one is created by the running. However, this is not possible, since $m_{\nu_{11}} = 0$ requires either a strong hierarchy or a difference of π between the Majorana phases, both of which prevent the generation of another texture zero.

Thus, the number of two-zero textures which are at least marginally compatible with experimental data can be raised from 9 to 12 if one includes RG effects. Furthermore, table 1 shows that a number of textures with three zeros, none of which is allowed at low energies [10], should be possible as well.

3 Examples for Radiatively Created Texture Zeros

In this section, we give numerical examples for the radiative generation of texture zeros. The RG evolution from the GUT scale to low energies is calculated numerically, starting with the desired texture zeros in the neutrino mass matrix and suitable values for the other parameters which ensure that all low-energy observables are compatible with experiment. This time, we solve the complete set of coupled differential equations and take into account the threshold corrections arising when the singlet neutrinos are successively integrated out at different energies [39, 40]. The examples are intended as a proof of principle, and therefore we show only one particular set of parameters for each case rather than trying to determine the complete allowed region in parameter space. For the same reason, we do not consider higher-order corrections from MSSM thresholds [46, 47] or two-loop effects [48], for example, which could change the low-energy value of the solar angle and may make it necessary to adjust some of the initial values, but do not spoil the general viability of the scenario.

3.1 Zeros in the 12- and 13-Entries

The texture with $m_{\nu_{12}} = m_{\nu_{13}} = 0$ (pattern F_1 in the classification of [11]) implies either $m_1 = m_2 = m_3$ or $\theta_{12} = 0$ and is therefore not compatible with data unless these predictions are changed by quantum corrections. Both cases have been studied in the literature [49–51], and it has been found that both can be compatible with the LMA solution at low energy. Therefore, we do not give an example here.

3.2 Vanishing Neutrino Mixings at High Energy

The numerical analysis of cases where a vanishing 23-element is generated shows that the 12- and 13-elements often become very small, too. This prompts one to ask whether exactly vanishing mixings might be possible as well. As shown in the left column of figure 1, this is indeed the case. In this example, we choose a diagonal neutrino mass matrix at $M_{\mathrm{GUT}} = 10^{16}$ GeV and use the MSSM with $\tan\beta = 50$ and a SUSY breaking scale of $M_{\mathrm{SUSY}} = 1$ TeV, below which the SM is assumed to be valid as an effective theory. The mass of the lightest neutrino at low energy is about 0.2 eV. The Majorana phases are set to zero without loss of generality, since they can be absorbed by a redefinition of the neutrino fields for a diagonal mass matrix. The numerical values of the light neutrino masses and Yukawa couplings at the GUT scale are

$$m_\nu(M_{\mathrm{GUT}}) = \begin{pmatrix} 0.2902 & 0 & 0 \\ 0 & 0.3056 & 0 \\ 0 & 0 & 0.3434 \end{pmatrix} \mathrm{eV}, \quad Y_\nu(M_{\mathrm{GUT}}) = \begin{pmatrix} -0.04 & 0 & 0 \\ -0.01 & 0.71 & -0.02 \\ 0.004 & -0.37 & 0.93 \end{pmatrix}.$$

The masses of the singlet neutrinos are $M_1 \approx 2 \cdot 10^{11}$ GeV, $M_2 \approx 3 \cdot 10^{13}$ GeV, and $M_3 \approx 10^{14}$ GeV. At M_Z, we obtain $\theta_{12} \approx 32°$, $\theta_{13} \approx 0.5°$, $\theta_{23} \approx 46°$, $\delta = 0$, $\Delta m_\odot^2 \approx 7.6 \cdot 10^{-5}$ eV^2, and $\Delta m_a^2 \approx 2.3 \cdot 10^{-3}$ eV^2, all well within the experimentally allowed 2σ ranges. For the neutrino mass matrix at the lowest see-saw scale, we find

$$m_\nu(M_1) = \begin{pmatrix} 0.2558 & 1.2 \cdot 10^{-4} & -4.0 \cdot 10^{-5} \\ 1.2 \cdot 10^{-4} & 0.2600 & 0.0041 \\ -4.0 \cdot 10^{-5} & 0.0041 & 0.2774 \end{pmatrix} \mathrm{eV} . \tag{19}$$

It clearly satisfies the requirement of a strong hierarchy between those elements that vanish at the GUT scale and the other entries, which we derived in section 2.2. Especially the large θ_{23} at low energies is easier to obtain in the MSSM than in the SM, since in the former the contribution of the running below the see-saw scale is also significant, if $\tan\beta$ is not too small, and drives the angle in the desired direction. Nevertheless, working examples can be found in the SM as well. Obviously, this result implies that the two-zero textures with $m_{\nu_{12}} = m_{\nu_{23}} = 0$ and $m_{\nu_{13}} = m_{\nu_{23}} = 0$ are possible as well.

One can see from the numerical precision of the above m_ν and Y_ν at the GUT scale that some tuning is necessary to obtain acceptable results. However, this is not that surprising considering that one needs an $\mathcal{O}(10^{-5}$ eV$^2)$ mass squared difference between mass eigenvalues whose squares are of the order of 10^{-2} eV^2. Furthermore, there is a lot of freedom to rearrange the values of the Yukawa couplings.
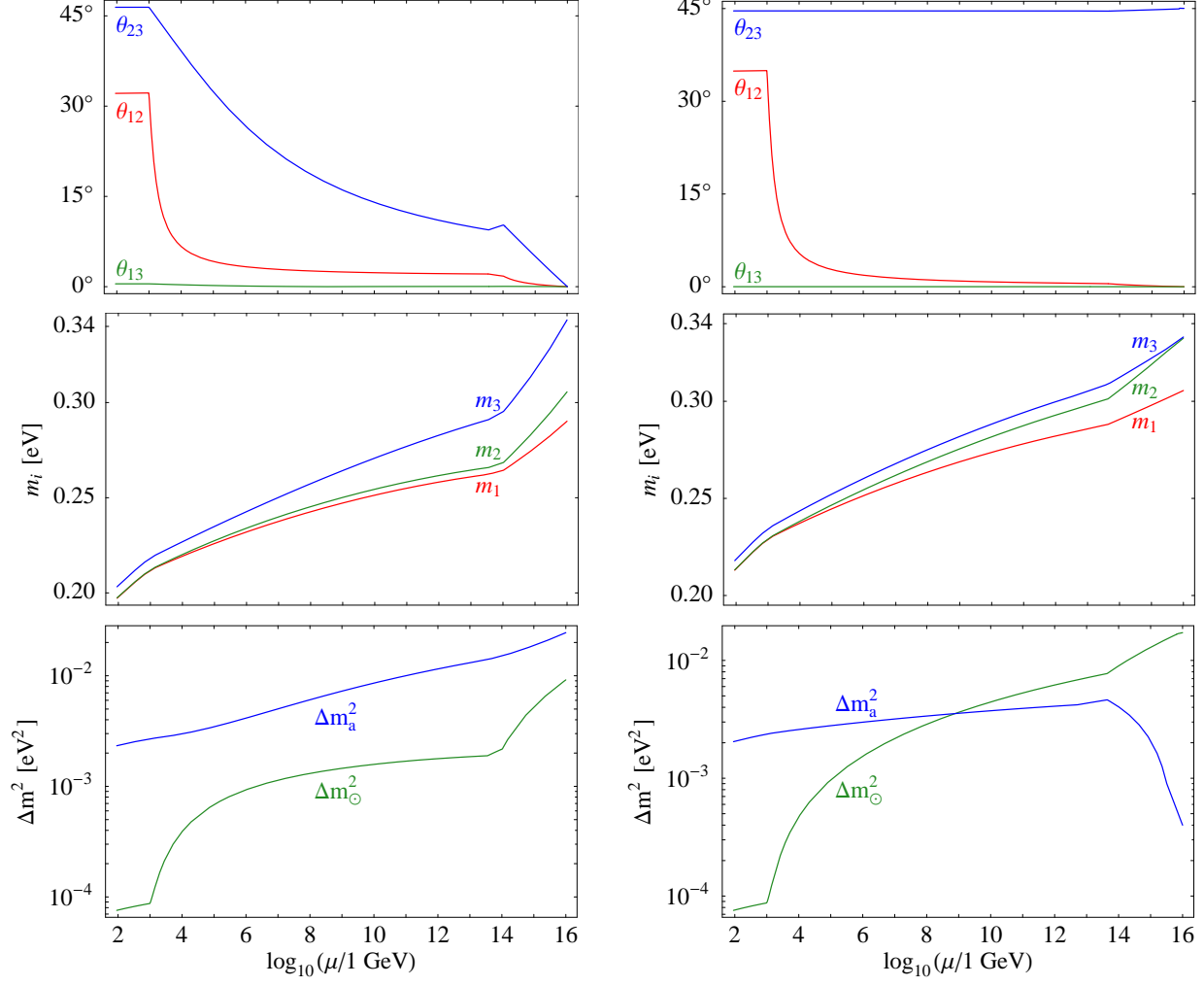
Figure 1: Running from vanishing neutrino mixing (left column) or a three-zero texture with $m_{\nu_{33}} = m_{\nu_{12}} = m_{\nu_{13}} = 0$ (right column) at the GUT scale $M_{\mathrm{GUT}} = 10^{16}$ GeV to the LMA solution at low energy. We used the MSSM with $\tan\beta = 50$ and a SUSY breaking scale $M_{\mathrm{SUSY}} = 1$ TeV. The kinks in the plots stem from integrating out the heavy singlets and the SUSY particles at their mass thresholds and the SUSY breaking scale, respectively. The mass eigenvalues are defined using the constant vacuum expectation value $v = v(0) \approx \sin\beta \cdot 246$ GeV.

3.3 Zeros in the 33-, 12- and 13-Entries

Finally, we consider an example where both Majorana phases equal π. As we derived in section 2, it should be possible to create zeros in the entries $m_{\nu_{22}}$, $m_{\nu_{33}}$, $m_{\nu_{12}}$, or $m_{\nu_{13}}$ in this case. The two-zero textures involving these elements (cases B and C in the classification of [10, 11]) are compatible with data anyway. Therefore, we pursue the more ambitious goal of creating a three-zero texture with $m_{\nu_{33}} = m_{\nu_{12}} = m_{\nu_{13}} = 0$. Again, we use the MSSM with $\tan\beta = 50$ and $M_{\mathrm{SUSY}} = 1$ TeV. The mass of the lightest neutrino at low energy is about 0.21 eV. The light neutrino mass matrix and the Yukawa couplings at the GUT scale are

$$m_\nu(M_{\mathrm{GUT}}) = \begin{pmatrix} -0.30554 & 0 & 0 \\ 0 & -0.0006 & 0.3328 \\ 0 & 0.3328 & 0 \end{pmatrix} \mathrm{eV}, \quad Y_\nu(M_{\mathrm{GUT}}) = \begin{pmatrix} -0.04 & 0 & -0.05 \\ 0.01 & 0.27 & 0 \\ 0.01 & -0.35 & 0.61 \end{pmatrix}.$$

The singlet neutrinos have the masses $M_1 \approx 10^{11}$ GeV, $M_2 \approx 5 \cdot 10^{12}$ GeV, and $M_3 \approx 4 \cdot 10^{13}$ GeV. For the oscillation parameters at low energy, we find $\theta_{12} \approx 35°$, $\theta_{13} \approx 0.01°$, $\theta_{23} \approx 45°$, $\delta = 0$, $\Delta m_\odot^2 \approx 7.6 \cdot 10^{-5}$ eV^2, and $\Delta m_{\mathrm{a}}^2 \approx 2.0 \cdot 10^{-3}$ eV^2. Their running is displayed in the right column of figure 1. In this example, the value of θ_{23} remains approximately 45° at all energies, since its RG evolution is strongly damped by the Majorana phases. At high energies, the mass ordering is inverted, but it changes to a normal hierarchy because the running of $\Delta m_\odot^2$ is very different from that of Δm_{a}^2. Of course, it is also possible to create a zero $m_{\nu_{22}}$ instead of $m_{\nu_{33}}$. However, both diagonal elements can vanish at the same time only if $m_{\nu_{12}}$ or $m_{\nu_{13}}$ remains finite.

4 Discussion and Conclusions

In this paper, the stability of zeros in neutrino mass matrices under quantum corrections has been studied and the consequences for the compatibility with experimental data have been discussed. We have considered a see-saw scenario with heavy singlet neutrinos where texture zeros in the effective mass matrix of the light neutrinos are assumed to be explained at the GUT scale. The discussion was performed in the basis where the mass matrix of the charged leptons is diagonal. The contributions of the off-diagonal elements of the neutrino Yukawa couplings to the RG running of the neutrino mass matrix can then replace the texture zeros by non-vanishing entries. From a bottom-up perspective, we have called this radiative generation of texture zeros. The positions where texture zeros can be generated depend on the CP parities of the mass eigenstates. As a consequence of this analysis, we find that some textures for the neutrino mass matrix that have been classified as incompatible with experimental data are not excluded. We find that three of the six forbidden two-zero textures as well as several three-zero textures are allowed. We have also shown that the radiative generation of texture zeros is not possible for hierarchical neutrino masses or for Majorana phases significantly different from 0 and π. Hence, the usual classification of forbidden and allowed textures applies in these cases. Texture zeros in the 12- and 13-entries of the mass matrix can also be created for arbitrary Majorana phases as long as they are approximately equal. The validity of our results was demonstrated by numerical examples. We included even a case where acceptable low-energy mixings are generated starting from a fully diagonal effective neutrino mass matrix at the GUT scale.

One should keep in mind that the mass matrix of the light neutrinos at high energy scales is a secondary quantity derived from the neutrino Yukawa couplings and the Majorana mass matrix of the singlet neutrinos. A more complete theory of flavour with texture zeros would probably be based on some symmetry predicting zeros in these matrices. As there are many possibilities, see e.g. [12, 13, 20], the analysis is beyond the scope of this work and remains to be done in future studies. However, it is clear that a statement which was found to be stable under radiative corrections in the present context will not change as it does not depend on the origin of the zeros in the neutrino mass matrix. A change is more likely the other way round: as the requirement of texture zeros in the Yukawa couplings and Majorana mass matrix removes some free parameters, the radiative generation of texture zeros becomes harder. It does not necessarily become impossible, however, since zeros in these matrices can be generated radiatively as well. It is also possible that a flavour symmetry guarantees the changes in the positions of the zeros below the GUT scale to be small. This obviously depends on the details of the model.

Another aspect concerns flavour-violating decays of charged leptons such as $\mu \to e\gamma$. It should be possible to suppress these processes sufficiently by adjusting the Yukawa couplings in such a way that the relevant elements of $Y_\nu^\dagger Y_\nu$ are very small.

Finally, one could also ask if an originally allowed texture can become forbidden at low energy. In particular examples, this is certainly possible, but it can always be avoided by choosing smaller Yukawa couplings, a smaller mass of the lightest neutrino or different values of the Majorana phases, all of which are not very strongly restricted by experiment. Hence, a complete exclusion of an allowed texture is not possible. In other words, the RG evolution can make a forbidden texture allowed, but not vice versa.

Acknowledgments

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